Proportional Reasoning of Pre-service Middle Grades Teachers

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UGA STEM Institute 2013

Ratio and proportion: Traditional instruction versus Common Core State Standards

Traditional instruction:

- emphasis on rote computation procedures, e.g., cross-multiplication
- little opportunity to reason about quantities varying together in a proportional relationship

A robust understanding of proportional relationships requires an interconnected set of understandings about multiplicative relationships and how quantities vary together.

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Ratio and proportion: Traditional instruction versus Common Core State Standards

Common Core State Standards for Mathematics ask students to:

- reason about quantities varying together in a proportional relationship;
- use tables and drawn representations: double number lines and strip diagrams.

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Peach Punch Problem

To make Peach Punch:
Mix peach juice and grape juice in the ratio 3 to 2.
How much grape juice should you mix with 5 cups peach juice?

Different ways to solve the Peach Punch Problem

Mix peach juice and grape juice in the ratio 3 to 2.
How much grape juice should you mix with 5 cups peach juice?

Method 1: Traditional cross-multiplying

$$\frac{2}{3} = \frac{x}{5} \qquad 2 \cdot 5 = 3 \cdot x$$

$$x = \frac{10}{3}$$

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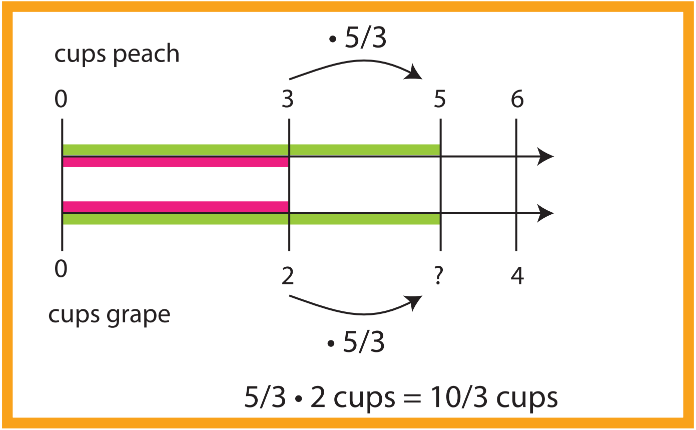
Different ways to solve the Peach Punch Problem

Mix peach juice and grape juice in the ratio 3 to 2.
How much grape juice should you mix with 5 cups peach juice?
Method 2: Double number line



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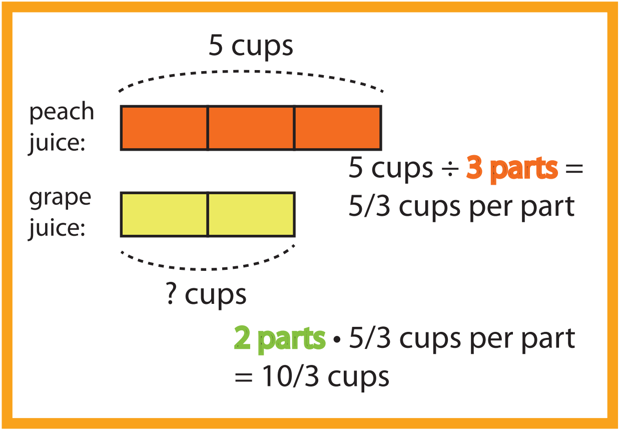
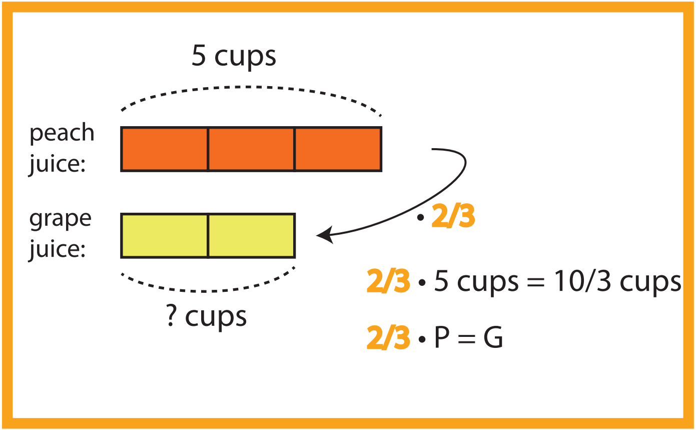
Different ways to solve the Peach Punch Problem

Mix peach juice and grape juice in the ratio 3 to 2.
How much grape juice should you mix with 5 cups peach juice?
Method 3: Strip diagram reasoning, parts



Different ways to solve the Peach Punch Problem

Mix peach juice and grape juice in the ratio 3 to 2.
How much grape juice should you mix with 5 cups peach juice?
Method 4: Strip diagram reasoning, multiplicative



Research on ratio and proportional relationships

There is a substantial body of research on children's thinking and learning about ratio and proportional relationships.

- Much is known about difficulties children have, e.g.,
  - attending to only one quantity,
  - comparing additively rather than multiplicatively,
  - applying proportions inappropriately,
  - not understanding ratios as measures of intensive quantities, ...
- Productive ways that children can reason are known, e.g.,
  - reasoning about coordinated values in tables, on double number lines

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Research on ratio and proportional relationships

Much less is known about pre-service and in-service teachers' thinking and learning about ratio and proportional relationships.

Different researchers have viewed ratio and rate in different ways.

Two types of proportional relationships

We have developed a mathematical analysis that reveals two perspectives/definitions of ratio and proportional relationships:

- “replicating batches”
  well-studied
- or “dilating parts”
  largely overlooked

These perspectives/definitions parallel the two quantitative perspectives/definitions for division:

- “how many in each group?” (partitive division)
- “how many groups?” (measurement division)

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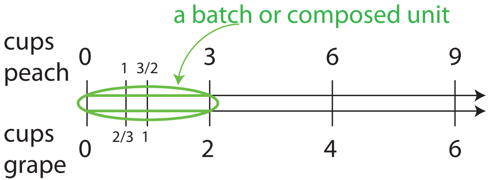
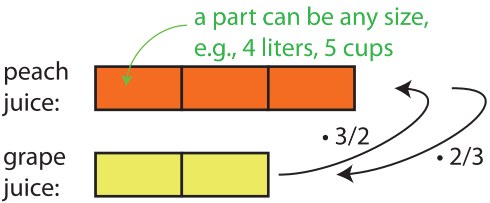
Peach and grape juice mixed in a 3 to 2 ratio



To make Peach Punch, mix 3 cups peach juice with every 2 cups grape juice.

batches	cups peach	cups grape	cups total
1	3	2	5
2	6	4	10
3	9	6	15

Peach and grape juice mixed in a 3 to 2 ratio



To make Peach Punch, mix 3 parts peach juice with 2 parts grape juice.

cups per part	cups peach	cups grape	cups total
1	3	2	5
2	6	4	10
3	9	6	15

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Why use the fixed numbers of variable parts perspective?

Potential advantages to the variable parts perspective:

- Better for showing visually the fixed multiplicative relationship between quantities;
- visually see how one quantity stretches to the other;
- better for developing equations using quantitative reasoning.

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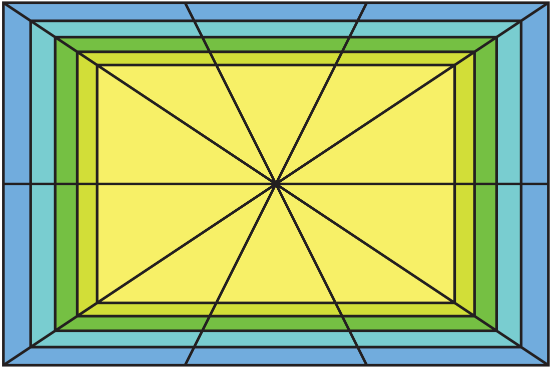
Why use the fixed numbers of variable parts perspective?

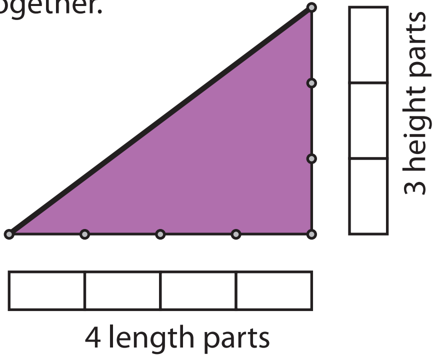
Rectangles whose height to width is in a 2 to 3 ratio:

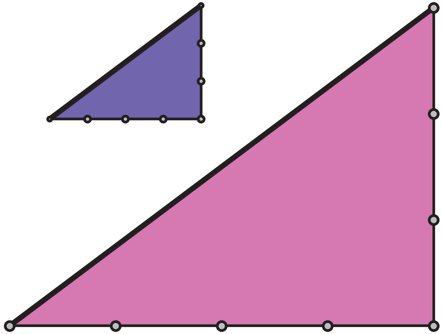


Why use the fixed numbers of variable parts perspective?

Ramps whose height to width is in a 3 to 4 ratio:

All parts are the same size and stretch or shrink together.





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The PRoMPT project

Proportional Reasoning of Middle Grades Pre-service Teachers

Funded by a UGA Office of STEM Education mini-grant

Other members of our research team:
Erik Jacobson
Eun Jung
Eun Kyung Kang
Burak Ölmez
Muhammet Arican

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Data

- Interviewed
  - 4 pairs of preservice middle school students
  - 4 pairs of preservice secondary teachers
  - 2 hour-long interviews with each pair
- All were in third semester of 3 semester sequences of content for teachers courses
  - number and operation
  - geometry
  - algebra
- Instruction emphasized:
  - the two perspectives on ratio
  - solving problems
  - reasoning with quantities
  - using drawn models.

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First Interview

Used mixture problems so that both perspectives could be applied

- Which perspectives do preservice teachers opt to use?
- How do number lines and strip diagrams support their reasoning?
- Where are they more and less proficient?

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First Interview

Sample tasks:

Scenario: A fragrant oil was made by mixing 3 milliliters lavender oil with 2 milliliters rose oil.

- What other amounts of lavender oil and rose oil can be mixed to make mixture that has exactly the same fragrance?
- How much lavender oil and how much rose oil do you need to make 35 mL of mixture?

Second Interview

Focus on students' understandings of variable parts and strip diagrams

- Can students think about parts as variable?
- Are students able to use *how many in one group?* division

Second Interview

Sample tasks:

Scenario: 14-karat gold for jewelry is made by mixing pure gold with a metal alloy in the ratio 7 to 5.

Explain how to reason about a strip diagram to solve the following: How much pure gold and copper will you need to mix 65 grams of jewelry gold?

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Preliminary Results

Similarities Across Pairs

- Students could explain and use both perspectives on ratio, at least to some extent
- Most students tended to use the batch perspective in combination with the variable parts perspective:
  they sometimes entered the number of batches into the parts of their strip diagrams

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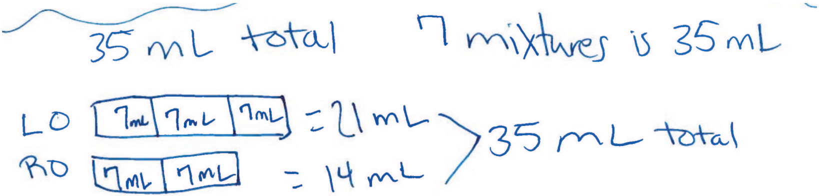
An Example

Mixing the “parts" and “batches” perspectives

How much lavender oil and how much rose oil do you need to make 35 mL of mixture?



Preliminary Results

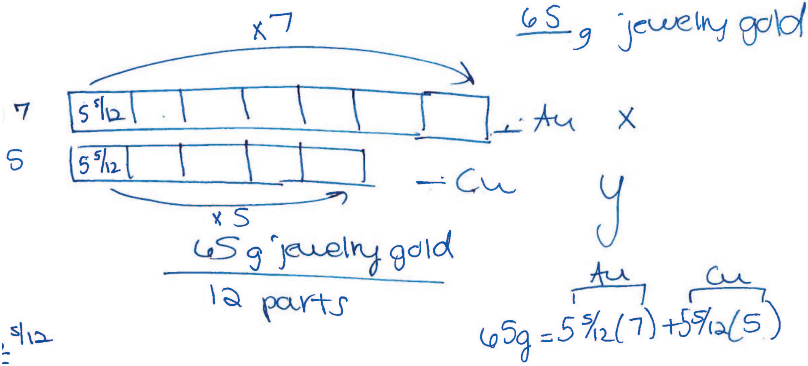
Divergence Across Pairs

- Some students tended to focus on additive relationships; others focused on multiplicative relationships
- Students' facilities with partitive division constrained their capacity to apply the "parts" perspective

An Example

Proficient reasoning with division and the “parts” perspective

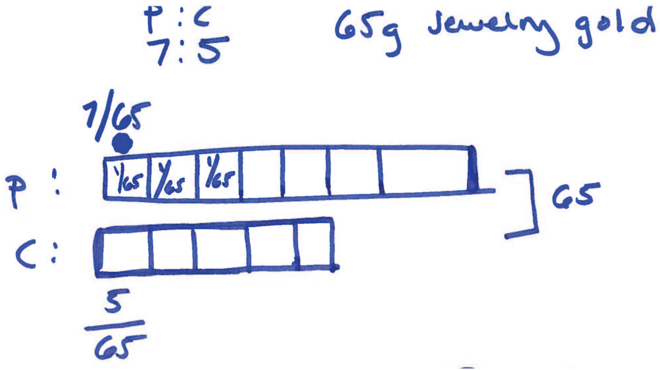
How much gold and how much copper do you need to make 65 g of jewelry gold mixed in a 7 to 5 ratio?



An Example

Inproficient reasoning with division and the “parts” perspective

How much gold and how much copper do you need to make 65 g of jewelry gold mixed in a 7 to 5 ratio?



Conclusions

- The two perspectives on ratio are accessible to preservice teachers
- Separating the reasoning associated with the two perspectives takes effort
- Applying the two perspectives on ratio pushes and reveals students' capacities with division

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